

Math 274: Tropical Geometry

UC Berkeley, Spring 2009

Homework # 1, due Tuesday, January 27

1. Prove the (tropical version of the) Fundamental Theorem of Algebra.
2. (*Tropical Cardano*) Derive a general formula, in terms of the three real coefficients a, b and c , for the roots of the tropical cubic polynomial:

$$f(x) = x^3 \oplus a \odot x^2 \oplus b \odot x \oplus c.$$

3. Given five general points in \mathbb{R}^2 , there exists a unique tropical quadric passing through these points. Compute and draw the quadratic curve passing through the points $(0, 5)$, $(1, 0)$, $(4, 2)$, $(7, 3)$ and $(9, 4)$.
4. A tropical cubic curve in \mathbb{R}^2 is *smooth* if it has precisely nine nodes. Prove that every smooth cubic curve has a unique bounded region, and that this region can have either three, four, five, six, seven, eight, or nine edges. Draw examples for all seven cases.
5. Determine the eigenvalue and all eigenvectors of the tropical matrix

$$A = \begin{pmatrix} 4 & 4 & 5 \\ 1 & 3 & 2 \\ 1 & 3 & 4 \end{pmatrix}.$$

What is the determinant of this matrix, and what is its image in \mathbb{TP}^2 ?

6. Determine and draw (in \mathbb{TP}^3) the images of the two matrices

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

7. Install Anders Jensen's software **GFan** on your computer. Read *Section 4: Doing tropical computations* of the manual and try an example.